MA 234, Spring 2016

Assignment 9

This homework is due Friday April 8.

There are total 45 points in this assignment. 40 points is considered 100%. If you go over 40 points, you will get over 100% for this homework (but not over 115%) and it will count towards your course grade.

Collaboration is welcome. If you do collaborate, make sure to write/type your own paper *and give credit to your collaborators in your pledge.* Your solutions should exhibit your work and contain full proofs. Bare answers will not earn you much.

This assignment covers Sections 6.2–6.3 (mostly 6.2) of Textbook.

(1) [10pt] Evaluate $\int_C y dz$ from $-3i$ to $3i$ along the following contours:
   (a) The right half of the circle $|z| = 3$.
   (b) The polygonal path C with vertices $-3i$, $3 - 3i$, $3 + 3i$, and $3i$.

(2) [10pt] Same question about the integral $\int_C z dz$.

(3) [10pt] By $C_r^+(a)$ we denote a circle of radius r centered at a traversed counterclockwise. By $C_r^-(a)$ we denote the same circle traversed clockwise. Evaluate the following integrals by a direct computation (not using Cauchy–Goursat Theorem, even when it's applicable).

   (a) $\int_{C_4^+(0)} z dz$.
   (b) $\int_{C_4^+(0)} \bar{z} dz$.
   (c) $\int_{C_2^+(0)} \frac{1}{z} dz$.
   (d) $\int_{C_2^-(0)} \frac{1}{z} dz$.
   (e) $\int_{C_2^-(0)} (1/\bar{z}) dz$.
   (f) $\int_{C_3^+(0)} \frac{1}{z^2} dz$.
   (g) $\int_{C_3^+(0)} (1/\bar{z}^2) dz$.

(4) [8pt] Use ML-inequality to show the following:
   (a) $\left| \int_C \frac{dz}{z^2 - 1} \right| \le \frac{\pi}{3}$, where C is the portion of $C_2^+(0)$ in the first quadrant. (*Hint:* On C, $|z| = 2$.)
   (b) $\left| \int_{C_R^+(0)} \frac{\mathrm{Log}(z)}{z^2} dz \right| \le 2\pi \frac{\sqrt{(\ln R)^2 + \pi^2}}{R}$. (*Hint:* On $C_R^+(0)$, $|z| = R$.)

(5) [7pt] Determine the domain of analyticity for the following functions and evaluate $\int_{C_1^+(0)} f(z) dz$ using Cauchy–Goursat theorem (or Deformation of contour theorem).
   (a) $f(z) = \tan z$.
   (b) $f(z) = \frac{1}{z - \frac{1}{2}}$.
   (c) $f(z) = \frac{z}{2z^2 - 5z + 2}$. (*Hint:* $2z^2 - 5z + 2 = (2z - 1)(z - 2)$.)
   (d) $f(z) = \frac{e^z}{z^2 - iz + 6}$.